Software tool for the master production schedule conception based on the Capacitated Lot Sizing Problem

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Introduction

Production management is a set of decisions that must reflect a compromise between customers' satisfaction and production criteria such as cost, delay and quality. The goal of production management is to ensure the continued success of the firm. Industrial planning plays a central part in this one. The latter can be divided into three hierarchical decisional levels:

- the strategic level deals with long-term decisions such as opening or closing of factories or determining building sites. (horizon is more than eighteen months),
- the tactical level which deals with conception of several plans such as Master Production Schedule, stock policy on a mid term horizon (six to eighteen months),
- the operational level which relates to the daily scheduling of the workshops.

Judging from the literature, the tactical planning is composed of two plans: The Sales and Operations Planning (S&OP) and the Master Production Schedule. The objective of S&OP is to obtain a compromise between sales objectives and production capacities. Therefore, it constrains the Master Production Schedule which determines, for each period, a balance between the capacity constraints and the customer's satisfaction while minimizing the production cost.

The mathematical model

Traditionally, the tactical planning models are based on Lot-Sizing models which determine the size of batches in order to minimize costs (setup cost, holding cost, production cost). Among those, there is a basic model: the "Capacitated Lot Sizing Problem" (CLSP) which elaborates the Master Production Schedule.

The CLSP can formally be described as a mixed-programming model:

$$Min \sum_{t=1}^{T}\left[\sum_{i=1}^{N}\left(s_i X_{it} + h_i I_{it} + r_i Q_{it}\right)\right] \qquad (Eq.1)$$

$s.c:$

$$\begin{cases} I_{i,t} = I_{i,t-1} + Q_{i,t} - d_{it}, & i \in \{1,..,N\}, \quad t \in \{1,..,T\} \quad (Eq.2) \\ \sum_{i=1}^{N} p_i Q_{it} \le C_t, & t \in \{1,..,T\} \quad (Eq.3) \\ Q_{it} \le C_t X_{it}, & i \in \{1,..,N\}, \quad t \in \{1,..,T\} \quad (Eq.4) \\ I_{it}, Q_{it} \in \mathbb{N}, & i \in \{1,..,N\}, \quad t \in \{1,..,T\} \quad (Eq.5) \\ Y_{it} \in \{0,1\}, & i \in \{1,..,N\}, \quad t \in \{1,..,T\} \quad (Eq.6) \end{cases}$$

<u>Parameters for the CLSP:</u>

N Number of items.
T Number of periods.
d_{it} External demand for item i at period t.
C_t Available capacity of the machine at period t.
p_i Capacity request for producing one unit of item i.
s_i Non-negative setup costs for item i.
h_i Non-negative holding costs for item i.
r_i Non-negative production costs for item i.
I_{i0} Initial inventory for item i.

<u>Decision variables for the CLSP:</u>

Q_{it} Production quantity of item i at period t.
I_{it} Inventory of item i at the end of period t.
X_{it} Binary variable which indicates whether a setup for item i occurs at period t ($X_{it}=1$) or not ($X_{it}=0$).

(Eq.1) is the objective function: it means the sum of the setup, the holding and the production costs that we seek to minimize.
(Eq.2) represents the inventory balances.
(Eq.3) represents the capacity constraint.
(Eq.4) represents the setup constraint: due to these restrictions, production of an item can only take place if the machine is set up for that particular item.
(Eq.5) are the non negativity conditions.
(Eq.6) the setup variables are defined as binary.

State of art

Solving CLSP is known as NP-Hard (Bitran et al. 1982). If positive setup times are added into the model, the feasibility problem is NP-complete (Trigeiro and al. 1989). In this case, the Eq 3 becomes

$$\sum_{i=1}^{N}\left(p_i Q_{it}+z_i X_{it}\right)\le C_t,\quad i\in\{1,..,N\},\quad t\in\{1,..,T\}\quad (Eq.3')$$

where z_i is the setup time to set the machine up to produce the item i at period t.

Many researchers have developed solutions for CSLPs, including mathematical programming (Leung et al 1997, Eppen et al. 1987, Belvaux et al. 2001 …), heuristic solutions (Dogramaci et al. 1981, Trigeiro et al. 1989, Diaby et al. 1993, Kirca et al. 1995, Degraeve et al. 2003 …), and metaheuristics (Gopalakrishnan et al. 2001, Özdamar et al. 2002, Karimi et al. 2005 …)

Our contribution

Our proposal is divided into two successive axes:

- The first one concerns the computation of a feasible solution, i.e. the design of a tactical plan which respects the capacity constraint and the customer's demand, even if setup times are considered,
- The second one is the cost optimization of the feasible solution found before.

In each step, we propose a metaheuristic. We encode the solution by a matrix representing a production plan. For each metaheuristic, we define an objective function and neighbouring systems.

Computation of a feasible solution:

So as to, we use the kangaroo algorithm, a metaheuristic based on simulated annealing (Fleury, 1993). For this one, we propose:

- a new quadratic objective function which models the capacity overshooting, whose minimization allows the smoothing of the production in order to find a feasible plan.
- two types of neighbouring systems: the first one allows to move a quantity period by period if and only if the customer's demand and capacity constraint are respected. The second one, changes the current solution, neglects capacity but respects demand.
- We use customer demand as initial solution for this method. In most of the cases, this solution is not feasible.

The quadratic function is defined as:

$$k(Q)=\sum_{t=1}^{T}\left[Max\left(0,\left(\sum_{i=1}^{N}p_i Q_{it}+z_i X_{it}\right)-C_t\right)\right]^2$$

where Q is the proposed planning. As Shown in Fig.1, this latter allows the smoothing of the production.

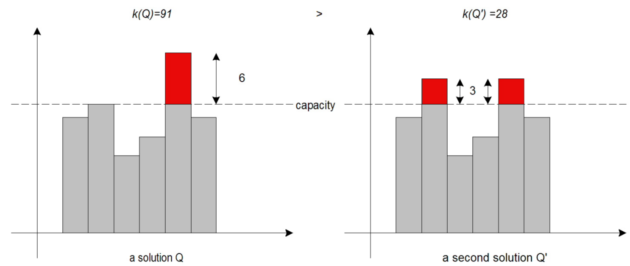

Fig1. Effect of the objective function

An initial solution Q^* is found if and only if $k(Q^*)=0$.

The first neighbouring system is summarised by the algorithm Algo1 :

Input: A solution Q
Output: A new solution Q^*
1. $Q^* \leftarrow Q$
2. Choose i randomly in {1,..,N}.
3. Choose randomly t_{start} *and* t_{target} *in {1..T}*
4. Compute K_{max}*, the maximal quantity of item i to be shifted from* t_{start} *to* t_{target} *according to proposition 1 below.*
5. Set α*:=0.6, choose randomly* $\beta\in[0,1]$
6. If $\beta\le\alpha$ *then choose randomly K in* $\{0,K_{max}\}$
else $K=K_{max}$.
7. $Q^*[i,t_{start}]:= Q^*[i,t_{start}]-K$
8. $Q^*[i,t_{target}]:= Q^*[i,t_{target}]+K$.

Algo1: Algorithm for the first neighbouring system.

α reflects distribution of two strategies: the $K=K_{max}$ strategy which tries to remove a maximum of setup time, and the other one which allows smaller adjustments.

Proposition1: Let Q a production planning which respects the demand. Q^* is another one if and only if:

- if target≤start:

$$K_{max}=\min\left\{\left\lfloor\left(C_{target}-\sum_{j=1}^{N}\left(p_j Q_{jtarget}-z_j X_{jtarget}\right)\right)/p_i\right\rfloor, Q_{istart}\right\}$$

- if start>target:

$$K_{max}=\min\left\{\left\lfloor\left(C_{target}-\sum_{j=1}^{N}\left(p_j Q_{jtarget}-z_j X_{jtarget}\right)\right)/p_i\right\rfloor, Q_{istart}, \min_{start\le t<target} I_{it}\right\}$$

The second neighbouring system uses the same algorithm but neglects capacity in the determination of K_{max}. By the way, we accept worse transitions in term of capacity.

Optimization of the feasible solution:

Concerning the optimization part, we use several metaheuristics based on simulated annealing algorithm:

- The objective function stands for the whole production cost for the plan as modelled in the CLSP (Eq 1),
- The neighbouring system allows the quantity moves period by period if and only if it respects the customer's demand and doesn't exceed the capacity of the target period. Moreover, it examines the inventory position in order to improve the computation of a new current solution while banishing bad tries.

The neighbouring system is based on the (Algo1). Indeed, proposition 1 ensures that if Q is feasible for the CLSP then Q^* deduced from (Ago1.) will keep that property. The major change is in the choice of α. Indeed, α is designed as a function of iteration count. Its goal is to support the choice of $K=K_{max}$ initially in order to remove a maximum of setup cost. This function converges gradually toward 0,6 because, after many tries, we have determined that it is a very good distribution for the two choices. Therefore

$$\alpha(iter)=0.6-1/\ln(1+10^3\times iter)$$

In order to measure the quality of the obtained solution, we have implemented a lagrangian relaxation to determine a lower bound. This one is inspired by Diaby's works (Diaby *et al.* 1993). Our relaxation deals with Capacities constraints (Eq.3). Indeed, Chen *et al.* 1990 have shown that it provides the best lower bound. The Wagner-Within algorithm (Wagner *et al.* 1958) is used to solve subproblems optimally.

Our method is tuned according to the general formula which guides lagrangian coefficients:

$$\lambda_t^{k+1} = Max\left[0, \lambda_t^k + \gamma_k \frac{(Z-Z_k)\sum_{i=1}^{N}\left(p_i Q_{it}^k + z_i X_{it}^k\right) - C_t}{\sum_{t=1}^{T}\left(\sum_{i=1}^{N}\left(p_i Q_{it}^k + z_i X_{it}^k\right) - C_t\right)^2}\right]$$

where, Z is an upper bound of the lagrangian relaxation, Z_k is the value of objective function for the lagrangian relaxation at iteration k: it could be updated during algorithm proceeding. $\left(Q_{it}^k, X_{it}^k\right)$ is the solution of the lagrangian relaxation problem at iteration k and $0 < \gamma_k \le 2$.

Computational results:

Early results of testing our approach are promising. Indeed, we have tested these different methods on small instances. For each instance, we obtained an optimal solution. We are testing these different methods on Trigeiro's instances which are commonly used like benchmark in literature. We chose the ones selected by Wolsey in his lotesizelib (2000) because they appear to be among the most difficult ones (Belvaux *et al.* 2000)

Tab.1 shows some results about feasibility for these instances:

Name	N. Items	N. periods	Success	N. Iter
G30	6	15	Y	14994
G53	12	15	Y	115336
G57	24	15	Y	4475
G62	6	30	Y	4265
G69	12	30	Y	10650
G72	24	30	Y	9049

Tab.1: Feasibility tests

As we say, we obtain very good results for the feasibility problem. We also test our optimization metaheuristic on these same instances neglecting setup time, in a first time.

Tab.2 shows some results about it:

Name	Solver	Kangaroo			
		Sol.	N. Iter	LB	Gap
G30	**37156***	37267	1651164	36939,5	0,89 %
G53	72831*	**71599**	1434315	70717,73	1,25 %
G57	137762*	**137659**	384215	135938,96	1,27 %
G62	**62058***	62545	1375810	60626,80	3,17 %
G69	132097*	**131839**	1224781	129589,97	1,74 %
G72	295291*	**295209**	910361	287390,70	2,73 %

Tab.2: Optimization Tests

We use Cplex 9.1 as solver for our tests: (*) means that it stops with an "overflow error". We test our metaheuristic with 600s time limit. We calculate our Lower Bound (LB) according to our lagrangean relaxation. We can see that we obtain very good results even if metaheuristic time limit is very weak. However, we can observe that higher is the number of items, better is our Kangaroo, compared to the Solver. To conclude, this approach is promising. We are still testing it on all Trigeiro's instances.

The software tool:

In order to test all these methods, we have designed a software tool which incorporates all these techniques and which allows us to follow, in real time, the evolution of the metaheuristics proposed. This software allows us to import the Trigeiro's instances or to create ours by using a convivial interface where we can parameterize the production system (capacity, cost …) and the customer's demand.

The main part of it is the optimization tool in which are implemented all the optimization methods seen before. On the GUI, we can follow the metaheuristic behavior, the gap of the best solution found (with the lower bound determinated by our lagrangian relaxation), and the best plan found, at any time of the optimization.

Our prospect:

We have proposed a software tool for the deterministic CLSP. However, within the framework of industrial planning, the managers work with estimated demands, thus potentially subjected to strong fluctuations. Therefore, we wish to extend our research by taking into account the uncertainty in the demand and the production capacities (resources into breakdowns etc.) so as to incorporate a new dimension: the robustness, in the obtained solution. Moreover, the presented model is a single-level model which doesn't take into account the material requirement planning (MRP) for the end items' planning conception. Moreover, it doesn't integrate the multi-sites aspect in the current productions systems. Therefore we are considered multi-sites models of planning based on the lot-sizing model: Multi Level Capacitated Lot Sizing Problem (MLCLSP). The next step would be to propose a new tactical planning's model, multi-levels and multi-sites as well as resolution approaches which could provide us a robust production plan, under an uncertain industrial context.

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